

The value of B_K from the experimental data on CP-violation in K -mesons and up-to-date values of CKM matrix parameters.

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Abstract

The difference between induced by box diagram quantity $\tilde{\epsilon}$ and experimentally measured value of ϵ is determined and used to obtain the value of $\tilde{\epsilon}$ with high precision. Present day knowledge of CKM matrix elements (including B-factory data), allows us to obtain from the Standard Model expression for $\tilde{\epsilon}$ the value of parameter B_K : $B_K = 0.89 \pm 0.16$. It turns out to be very close to the result of vacuum insertion, $B_K = 1$.

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1 Introduction.

It is well known that CP - violation in $K^0 - \bar{K}^0$ mixing is described by the parameter $\tilde{\epsilon}$. Within the SM, this parameter is given by box diagrams. It depends in particular on the CKM matrix elements, to which vertices of box diagrams are proportional. On the other hand, the experimentally measured parameters are ϵ and ϵ' . ϵ and ϵ' enter the measured ratios of decay amplitudes of kaons into $\pi\pi$ states. These amplitudes are superpositions of amplitudes $A(K^0 \rightarrow (\pi\pi)_I) = A_I e^{i\delta_I}$ of kaon decays into states with definite isospin $I = 0, 2$, A_I are weak amplitudes, δ_I are strong rescattering phases of π -mesons. The parameter ϵ can be expressed as [1]:

$$\epsilon = \tilde{\epsilon} + i \frac{ImA_0}{ReA_0}. \quad (1)$$

Within the SM and in the standard parametrization of CKM matrix, ImA_0 originates from the so-called strong penguin diagrams. Amplitude A_2 also has an imaginary part which originates from electro-weak penguin diagrams. That is why $ImA_0 \gg ImA_2$.

Taking into account that the phases of ϵ and $\tilde{\epsilon}$ are approximately $\frac{\pi}{4}$ [1], from Eq.(1) we obtain:

$$|\tilde{\epsilon}| \approx |\epsilon| - \frac{1}{\sqrt{2}} \frac{ImA_0}{ReA_0}. \quad (2)$$

The estimation of $\frac{ImA_0}{ReA_0}$ was done in [2]. This term appears to be a 5 – 9% correction to the value of $\tilde{\epsilon}$ in Eq.(2). Provided that we have estimated the right-hand side of Eq.(2) with the help of Eq.(13) we can determine parameter B_K , which parameterizes hadronic matrix element. Of course, for this purpose we need to know the values of CKM matrix elements that enter Eq.(13).

The parameters $\bar{\rho}$ and $\bar{\eta}$ of CKM matrix appear to be constrained without using the value of $\tilde{\epsilon}$ in the fit. Thus we perform the fit of CKM matrix parameters without using constraint from $\tilde{\epsilon}$ in it. Then we determine B_K from Eqs.(2),(13). Our result is $B_K = 0.89 \pm 0.16$.

This result is close to the result of vacuum insertion: $B_K = 1$. As discussed in [3], the insertions of π -mesons states should be taken into account. These insertions form a sign-alternating series, whose terms depend on the cutoff momentum of π -mesons. This cutoff can be reasonably chosen to be 200 – 500 MeV (at larger virtualities π -mesons do not exist). Then the sign-alternating series converges quickly, and one can take only first two terms. Thus taking into account the insertions of π -mesons states lowers B_K , and the agreement with our result improves further.

The lattice result of B_K calculation is $B_K = 0.87 \pm 0.06 \pm 0.14_{quench}$ [4]. We see that our result is very close to it.

The paper is organized as follows: in Section 2 we discuss various estimations of the value of $\frac{ImA_0}{ReA_0}$. In Section 3 we perform the fit of CKM matrix parameters without using constraint from $\tilde{\epsilon}$ in it. In Section 4 we determine B_K and compare it with other results of calculation of B_K . Finally, we make our conclusion in Section 5.

2 Estimation of the numerical value of $\epsilon - \tilde{\epsilon}$.

In this section we review the estimation of $\epsilon - \tilde{\epsilon}$ [2]. We discuss the following three methods. First, one can use the experimental data on CP-violation in semileptonic K_L -decays, namely parameter δ_L . This method possesses large uncertainty and also at the level of two sigmas contradicts the experimental value of $\frac{\epsilon'}{\epsilon}$. Second, one can obtain the lower bound on $\epsilon - \tilde{\epsilon}$ from the experimental value of $\frac{\epsilon'}{\epsilon}$. This lower bound is important in understanding the relative magnitude of the second term in Eq.(2), it turns out to be $\geq 5\%$. Third, we use the results of direct computation of ImA_0 in the ratio $\frac{ImA_0}{ReA_0}$, substituting the experimental value of ReA_0 . This gives us a reliable estimate of $\frac{ImA_0}{ReA_0}$ with moderate error, which we use in the bulk of the paper.

First, we estimate the value of $\tilde{\epsilon}$ from the experimental results on CP-violation in semileptonic K_L decays:

$$\delta_L = \frac{\Gamma(K_L \rightarrow l^+ \nu \pi^-) - \Gamma(K_L \rightarrow l^- \bar{\nu} \pi^+)}{\Gamma(K_L \rightarrow l^+ \nu \pi^-) + \Gamma(K_L \rightarrow l^- \bar{\nu} \pi^+)} \approx 2Re\tilde{\epsilon}.$$

$$|\tilde{\epsilon}| = \frac{\delta_L}{2 \cos \phi}, \quad (3)$$

where $\phi = arg(\tilde{\epsilon})$.

Now let us substitute the experimental data. For ϕ we use $\phi = (43.50 \pm 0.05)^\circ$ [5]. World average value of δ_L , published in [6], contains new KTeV result: $\delta_L = (3.307 \pm 0.063) \times 10^{-3}$. Na48 collaboration recently obtained: $\delta_L = (3.317 \pm 0.100) \times 10^{-3}$ [7]. Averaging these two numbers we get: $\delta_L = (3.310 \pm 0.053) \times 10^{-3}$. This leads to the following value of $\tilde{\epsilon}$:

$$|\tilde{\epsilon}| = (2.282 \pm 0.037) \times 10^{-3}. \quad (4)$$

From Eq.(2) with the help of Eq.(8) we can find the corresponding value of $\frac{ImA_0}{ReA_0}$:

$$\frac{ImA_0}{ReA_0} = (0.03 \pm 0.56) \times 10^{-4}. \quad (5)$$

We will show below, that this number almost contradicts the present experimental value of $\frac{\epsilon'}{\epsilon} = (1.67 \pm 0.26) \times 10^{-3}$ [5].

Second method of estimation of $\frac{ImA_0}{ReA_0}$, which gives the lower bound on it, uses the experimental value of $\frac{\epsilon'}{\epsilon}$. The expression for $\frac{\epsilon'}{\epsilon}$ is usually presented as follows [1]:

$$\frac{\epsilon'}{\epsilon} = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{1}{\epsilon} \left[\frac{ImA_2}{ReA_0} - w \frac{ImA_0}{ReA_0} \right], \quad (6)$$

Let us neglect the term proportional to ImA_2 in Eq.(6), which comes from the EW penguins. Taking into account that $(\delta_0 - \delta_2)_{exp} = 42 \pm 4^\circ$ [8], we obtain the following expression for $\frac{ImA_0}{ReA_0}$ from Eq.(6):

$$\frac{ImA_0}{ReA_0} \approx -\frac{\sqrt{2}|\epsilon|}{w} \frac{\epsilon'}{\epsilon}. \quad (7)$$

Substituting experimental values from [5], we get:

$$\begin{aligned} \frac{\epsilon'}{\epsilon} &= (1.67 \pm 0.26) \times 10^{-3}, \quad w = 0.045, \quad |\epsilon| = 2.284(14) \times 10^{-3} \implies \\ \frac{ImA_0}{ReA_0} &= -(1.2 \pm 0.2) \times 10^{-4}. \end{aligned} \quad (8)$$

In this way we get the following value of $|\tilde{\epsilon}|$:

$$|\tilde{\epsilon}| = 2.37(2) \times 10^{-3}. \quad (9)$$

Since, according to Eq.(6), the contribution of EW penguins partially cancels that of QCD penguin, the value $|\frac{ImA_0}{ReA_0}| = (1.2 \pm 0.2) \times 10^{-4}$ should be considered as a lower bound on $|\frac{ImA_0}{ReA_0}|$ and Eq.(9) is a lower bound on $|\tilde{\epsilon}|$. Thus the central value of $\frac{ImA_0}{ReA_0}$, obtained from semileptonic K_L -decays, Eq.(5), almost contradicts the experimental value of $\frac{\epsilon'}{\epsilon}$.

Finally, the reliable way to estimate $\frac{ImA_0}{ReA_0}$, result of which we will use in the next sections is to use the experimental value of ReA_0 and theoretical value for ImA_0 .

Calculation of ReA_0 and ImA_0 , as well as ReA_2 and ImA_2 , has a long history. The calculation of ReA_0 and ReA_2 was performed in order to explain the $\Delta I = \frac{1}{2}$ rule in kaon decays and the calculation of ImA_0 and ImA_2 - in order to explain the observed value of $\frac{\epsilon'}{\epsilon}$.

In this paper we perform the calculation of ImA_0 to the following accuracy: the Wilson coefficient is calculated to LO and hadronic matrix element is calculated in naive factorization approximation. The details are presented in Appendix, and here we only quote the result:

$$\frac{ImA_0}{ReA_0} = -(3.2^{+1.1}_{-0.8}) \times 10^{-4}. \quad (10)$$

We note that the results of computation of ImA_0 (see [9] - [14] and refs. therein) performed by a large number of people lie in the same ballpark.

Finally, from Eq.(10) we can determine the value of $\tilde{\epsilon}$:

$$|\tilde{\epsilon}| = (2.51 \pm 0.07) \times 10^{-3}. \quad (11)$$

This number is our final result, and we will use it in Section 4.

3 Fit of the parameters of CKM matrix

We use in our fit of the CKM matrix experimentally measured values of modulus of matrix elements $V_{ud}, V_{us}, V_{ub}, V_{cd}, V_{cs}, V_{cb}$ and also $\sin 2\alpha, \sin 2\beta, \sin 2\gamma$ and Δm_{B_d} . Note that we

do not use $\tilde{\epsilon}$ in fit, since we plan to determine the value of B_K with the help of the fit results.

We assume these experimentally measured data to be normally distributed. Also the theoretical uncertainties are treated as normally distributed. Let us note that other people treat theoretical uncertainties in other way [16], [17].

The table of input parameters looks like:

Parameter	Value	Standard Deviation
$ V_{ud} $ [5]	0.9738	0.0005
$ V_{us} $ [5]	0.2200	0.0026
$ V_{ub} $ [5]	0.00367	0.00047
$ V_{cd} $ [5]	0.224	0.012
$ V_{cs} $ [5]	0.996	0.013
$ V_{cb} $ [5]	0.0413	0.0015
$\sin 2\alpha$ [18]	-0.21	0.46
$\sin 2\beta$ [5]	0.736	0.049
$\sin 2\gamma$ [19]	0.69	0.58

The χ^2 expression which we minimize looks like:

$$\begin{aligned}
\chi^2(A, \lambda, \bar{\rho}, \bar{\eta}) = & \left(\frac{V_{ud}^{theo} - V_{ud}^{exp}}{\sigma_{V_{ud}}} \right)^2 + \left(\frac{V_{us}^{theo} - V_{us}^{exp}}{\sigma_{V_{us}}} \right)^2 + \left(\frac{V_{ub}^{theo} - V_{ub}^{exp}}{\sigma_{V_{ub}}} \right)^2 + \left(\frac{V_{cd}^{theo} - V_{cd}^{exp}}{\sigma_{V_{cd}}} \right)^2 + \\
& + \left(\frac{V_{cs}^{theo} - V_{cs}^{exp}}{\sigma_{V_{cs}}} \right)^2 + \left(\frac{V_{cb}^{theo} - V_{cb}^{exp}}{\sigma_{V_{cb}}} \right)^2 + \left(\frac{\Delta m_{B_d}^{theo} - \Delta m_{B_d}^{exp}}{\sigma_{\Delta m}} \right)^2 + \left(\frac{\sin 2\alpha^{theo} - \sin 2\alpha^{exp}}{\sigma_{\sin 2\alpha}} \right)^2 + \\
& + \left(\frac{\sin 2\beta^{theo} - \sin 2\beta^{exp}}{\sigma_{\sin 2\beta}} \right)^2 + \left(\frac{\sin 2\gamma^{theo} - \sin 2\gamma^{exp}}{\sigma_{\sin 2\gamma}} \right)^2, \tag{12}
\end{aligned}$$

where theoretical expressions depend on four Wolfenstein parameters: A , λ , $\bar{\rho}$ and $\bar{\eta}$. Expression (12) was minimized varying them.

Here are our results:

$$\begin{aligned}
\lambda &= 0.224 \pm 0.002 & \alpha^{[deg]} &= 100 \pm 5 \\
A &= 0.82 \pm 0.03 & \beta^{[deg]} &= 23 \pm 2 \\
\bar{\rho} &= 0.22 \pm 0.04 & \gamma^{[deg]} &= 57 \pm 5 \\
\bar{\eta} &= 0.34 \pm 0.02 \\
\chi^2/n.d.o.f. &= 8.1/5.
\end{aligned}$$

For comparison, we present the results of the fit, made by CKMfitter Group [16] and UTfit Collaboration [17]:

	CKMfitter	UTfit
λ	0.226 ± 0.002	0.226 ± 0.002
A	$0.80^{+0.03}_{-0.02}$	
$\bar{\rho}$	$0.19^{+0.09}_{-0.07}$	0.17 ± 0.05
$\bar{\eta}$	$0.36^{+0.05}_{-0.04}$	0.35 ± 0.03

	CKMfitter	UTfit
$\alpha^{[deg]}$	94^{+12}_{-10}	94 ± 8
$\beta^{[deg]}$	$23.8^{+2.1}_{-2.0}$	23.2 ± 1.4
$\gamma^{[deg]}$	62^{+10}_{-12}	61.6 ± 7

4 The value of B_K

From the results of the fit, presented above, we can extract the value of B_K . For this purpose we use the theoretical expression for $|\tilde{\epsilon}|$, first obtained in [20]. It has the following form:

$$|\tilde{\epsilon}^{theo}| = \frac{G_F^2 m_K f_K^2}{12\sqrt{2}\pi^2 \Delta m_K} B_K (\eta_{cc} m_c^2 \text{Im}[(V_{cs} V_{cd}^*)^2] + \eta_{tt} m_t^2 I(\xi) \text{Im}[(V_{ts} V_{td}^*)^2] + 2\eta_{ct} m_c^2 \ln(\frac{m_W^2}{m_c^2}) \text{Im}[V_{cs} V_{cd}^* V_{ts} V_{td}^*]). \quad (13)$$

Here $I(\xi) = \{\frac{\xi^2 - 11\xi + 4}{4(\xi - 1)^2} - \frac{3\xi^2 \ln \xi}{2(1 - \xi)^3}\}$, $\xi = m_t^2/m_W^2$. Quark masses are $m_c = 1.2 \pm 0.2$ GeV [5], $m_t = 178.0 \pm 4.3$ GeV [21], $m_W = 80.42 \pm 0.04$ GeV [5]. The QCD corrections were calculated to leading order in [20]: $\eta_{cc} = 0.6$, $\eta_{tt} = 0.6$, $\eta_{ct} = 0.4$. The next-to-leading order calculation changes slightly η_{tt} and η_{ct} and changes considerably η_{cc} : $\eta_{cc} = 1.32 \pm 0.32$ [22], $\eta_{tt} = 0.574 \pm 0.01$ [23], $\eta_{ct} = 0.47 \pm 0.04$ [24]. The kaon decay constant extracted from the $K^+ \rightarrow \mu^+ \nu$ decay width equals: $f_K = 160.4 \pm 1.9$ MeV [5]. The $K_L - K_S$ mass difference is $\Delta m_K = (3.483 \pm 0.006) \times 10^{-15}$ GeV [5]. Fermi constant $G_F = 1.16639(1) \times 10^{-5} \text{GeV}^{-2}$ [5].

Now we equate this expression to the value of $|\tilde{\epsilon}| = (2.51 \pm 0.07) \times 10^{-3}$ from Eq.(11), substituting all experimental numbers and the results of the fit. This leads to the following value of B_K :

$$B_K = 0.89 \pm 0.16 \quad (14)$$

Note that it is close to the result of vacuum insertion: $B_K = 1$.

5 Conclusions

We have extracted the value of B_K using the fitted values of CKM matrix elements and the estimated difference between $\tilde{\epsilon}$ and ϵ . Our result is $B_K = 0.89 \pm 0.16$. It appears to be close to the result of vacuum insertion, $B_K = 1$, while lattice result is simply the same: $B_K = 0.87 \pm 0.06 \pm 0.14_{\text{quench}}$ [4].

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A Estimation of the value of $\text{Im}A_0$ from QCD penguin diagram.

Let's estimate $\frac{\text{Im}A_0}{\text{Re}A_0}$, using experimental value of $\text{Re}A_0$ and evaluating the value of $\text{Im}A_0$. The latter will be evaluated to the following accuracy: the LO Wilson coefficients will be

used, and hadronic matrix element will be calculated in naive factorization approximation.

As it is well known transitions with $\Delta S = 1$ are due to the 4-quark effective Hamiltonian, for the first time derived in [25]:

$$H_{\Delta S=1} = \sqrt{2}G_F \sin \theta_C \cos \theta_C \sum_{i=1}^6 c_i O_i \quad (15)$$

The so-called penguin operator O_5 dominates in amplitudes $K^0 \rightarrow (\pi\pi)_{I=0}$ [25]:

$$O_5 = \bar{s}_L \gamma_\mu \lambda^a d_L (\bar{u}_R \gamma_\mu \lambda^a u_R + \bar{d}_R \gamma_\mu \lambda^a d_R) \quad (16)$$

Below we present the detailed derivation of the coefficient function c_5 in one loop approximation.

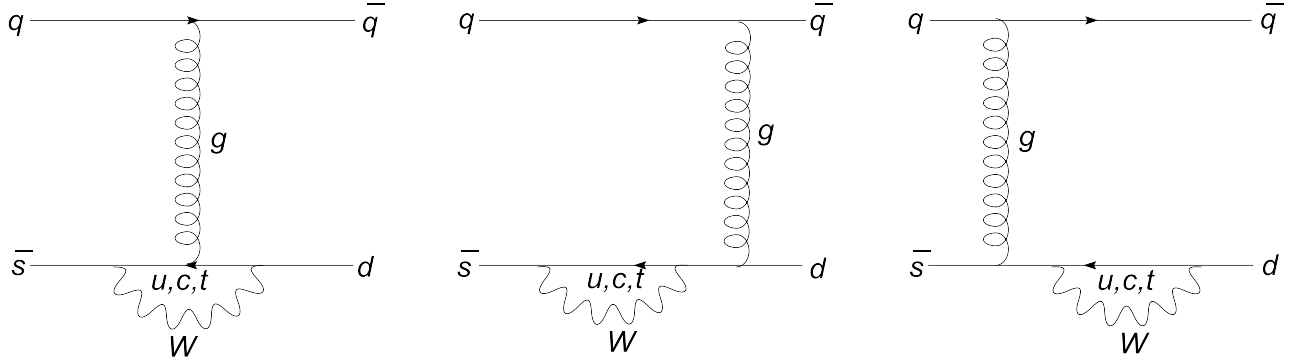


Figure 1: *Diagrams, which contribute to the penguin operator.*

It is convenient to perform calculation in the unitary gauge. Each of the three diagrams (see Fig. 1) is infinite, however the sum appears to be finite. We will use the dimensional regularization ($d = 4 - 2\epsilon$), in order to regularize divergent integrals:

$$\begin{aligned} M_0 &= \frac{g^2 g_s^2}{2} \frac{1}{q^2} \sum_{j=u,c,t} \int \frac{d^d k}{(2\pi)^d} \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2} \bar{s}_L \gamma_\mu \frac{1}{\hat{q}_2 - m_j} \gamma_\rho \frac{\lambda_a}{2} \frac{1}{\hat{q}_1 - m_j} \gamma_\nu d_L \left(\bar{\psi}_r \gamma_\rho \frac{\lambda_a}{2} \psi_r \right) V_{jd} V_{js}^*, \\ M_1 &= \frac{g^2 g_s^2}{2} \frac{1}{q^2} \sum_{j=u,c,t} \int \frac{d^d k}{(2\pi)^d} \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2} \bar{s}_L \gamma_\mu p_L \frac{1}{\hat{q}_2 - m_j} \gamma_\nu p_L \frac{1}{\hat{p}_2 - m_j} \gamma_\rho \frac{\lambda_a}{2} d \left(\bar{\psi}_r \gamma_\rho \frac{\lambda_a}{2} \psi_r \right) V_{jd} V_{js}^*, \\ M_2 &= \frac{g^2 g_s^2}{2} \frac{1}{q^2} \sum_{j=u,c,t} \int \frac{d^d k}{(2\pi)^d} \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2} \bar{s}_L \gamma_\rho \frac{\lambda_a}{2} \frac{1}{\hat{p}_1 - m_j} \gamma_\mu p_L \frac{1}{\hat{q}_1 - m_j} \gamma_\nu p_L d \left(\bar{\psi}_r \gamma_\rho \frac{\lambda_a}{2} \psi_r \right) V_{jd} V_{js}^*, \\ M &= M_0 + M_1 + M_2. \end{aligned} \quad (17)$$

The effective Hamiltonian is equal to:

$$H_{\text{penguin}} = iM. \quad (18)$$

The result of the calculation can be presented in the following form:

$$H_{penguin} = - \sum_{j=u,c,t} \frac{g^2 g_s^2}{48\pi^2} \frac{1}{q^2} \bar{s}_L G_\rho(q) \frac{\lambda_a}{2} d_L \left(\bar{\psi}_r \gamma_\rho \frac{\lambda_a}{2} \psi_r \right) V_{jd} V_{js}^*,$$

$$G_\rho = G_1 \gamma_\rho + \frac{1}{M_W^2} (G_2 \hat{p}_2 \gamma_\rho \hat{p}_1 + G_3 \hat{p}_1 p_{1\rho} + G_4 \hat{p}_2 p_{2\rho} + G_5 \hat{p}_1 p_{2\rho} + G_6 \hat{p}_2 p_{1\rho}). \quad (19)$$

Dimensionless functions G_i ($i = 1, 2, 3, 4, 5, 6$) depend on the values $x_j = \frac{m_j^2}{M_W^2}$, q^2 , m_s^2 , m_d^2 , where m_j is the up-quark mass in the loop. We suppose the external s and d quarks to be on mass shell.

Let us neglect d -quark mass. In this approximation the non-zero contribution into operator $H_{penguin}$ is given by the terms with formfactors G_1, G_4 and G_6 . It is convenient to introduce new variables $P = p_1 + p_2$ and $q = p_1 - p_2$:

$$\bar{s}_L G_\rho \frac{\lambda_a}{2} d_L = G_1 \bar{s}_L \gamma_\rho \frac{\lambda_a}{2} d_L + \frac{m_s ((G_6 + G_4) P_\rho + (G_6 - G_4) q_\rho)}{2M_W^2} \bar{s}_R \frac{\lambda_a}{2} d_L. \quad (20)$$

Here the term proportional to q_ρ will not contribute to operator $H_{penguin}$, since $q_\rho \times (\bar{\psi}_r \gamma_\rho \frac{\lambda_a}{2} \psi_r) = (m_r - m_r) (\bar{\psi}_2 \frac{\lambda_a}{2} \psi_1) = 0$. The quantity $P_\rho \bar{s}_R \frac{\lambda_a}{2} d_L$ should be expressed through the magnetic formfactor with the help of the following equation:

$$\bar{s} \sigma_{\mu\nu} q_\nu \frac{\lambda_a}{2} d_L = \frac{i}{2} q_\nu \bar{s} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \frac{\lambda_a}{2} d_L = i(m_s \bar{s}_L \gamma_\mu \frac{\lambda_a}{2} d_L - P_\mu \bar{s}_R \frac{\lambda_a}{2} d_L). \quad (21)$$

With the help of Eq.(21) from Eq.(20) we obtain:

$$H_{penguin} = - \sum_{j=u,c,t} \frac{g^2 g_s^2}{48\pi^2} \frac{1}{q^2} \left(f_1 \bar{s}_L \gamma_\mu \frac{\lambda_a}{2} d_L + i \frac{f_2}{M_W^2} m_s q_\nu \bar{s}_R \sigma_{\mu\nu} \frac{\lambda_a}{2} d_L \right) \bar{\psi}_r \gamma_\mu \frac{\lambda_a}{2} \psi_r V_{jd} V_{js}^*, \quad (22)$$

where f_1 and f_2 are equal to:

$$f_1 = G_1 + \frac{m_s^2}{2M_W^2} (G_6 + G_4),$$

$$f_2 = \frac{G_6 + G_4}{2}. \quad (23)$$

It is sufficient to calculate the formfactors G_4 and G_6 in the zero order in q^2 , m_s^2 . However, the formfactor G_1 , as it follows from last equations, should be calculated, including terms proportional to m_s^2 , q^2 . From equations (17), calculating appropriate integrals, we get:

$$G_1 = R_1 \frac{q^2}{M_W^2} + R_2 \frac{m_s^2}{M_W^2},$$

$$R_1 = \frac{7x^4 + 14x^3 - 63x^2 + 38x + 4 + 6(16x - 9x^2 - 4) \ln x}{24(1-x)^4},$$

$$R_2 = \frac{-5x^4 + 14x^3 - 39x^2 + 38x - 8 + 18x^2 \ln x}{8(1-x)^4},$$

$$\begin{aligned}
G_4 &= \frac{2x^4 - 14x^3 + 45x^2 - 38x + 5 + 6(1 - 4x) \ln x}{6(1 - x)^4}, \\
G_6 &= \frac{11x^4 - 14x^3 + 27x^2 - 38x + 14 + 6(8x - 9x^2 - 2) \ln x}{12(1 - x)^4}.
\end{aligned} \tag{24}$$

Substituting these formulas into equations (23), we obtain:

$$\begin{aligned}
f_1 &= \frac{7x^4 + 14x^3 - 63x^2 + 38x + 4 + 6(16x - 9x^2 - 4) \ln x}{24(1 - x)^4} \frac{q^2}{M_W^2}, \\
f_2 &= \frac{5x^4 - 14x^3 + 39x^2 - 38x + 8 - 18x^2 \ln x}{8(1 - x)^4}.
\end{aligned} \tag{25}$$

Finally, let us rewrite equation (22) in the following way:

$$\begin{aligned}
H_{penguin} &= - \sum_{j=u,c,t} \frac{g^2 g_s^2}{48\pi^2 M_W^2} \left(F_1 \bar{s}_L \gamma_\mu \frac{\lambda_a}{2} d_L + i F_2 m_s \frac{q_\nu}{q^2} \bar{s}_R \sigma_{\mu\nu} \frac{\lambda_a}{2} d_L \right) \bar{\psi}_r \gamma_\mu \frac{\lambda_a}{2} \psi_r V_{jd} V_{js}^*, \\
F_1 &= \frac{7x^4 + 14x^3 - 63x^2 + 38x + 4 + 6(16x - 9x^2 - 4) \ln x}{24(1 - x)^4}, \\
F_2 &= f_2 = \frac{5x^4 - 14x^3 + 39x^2 - 38x + 8 - 18x^2 \ln x}{8(1 - x)^4}.
\end{aligned} \tag{26}$$

As the admixture of gluons in K and π mesons is small, the contribution of magnetic moment operator in (26) is negligible [25].

Substituting $m_c = 1.2$ GeV, $m_t = 178.0$ GeV, $M_W = 80.42$ GeV for the formfactor F_1 we obtain:

$$\begin{aligned}
F_1(x \ll 1) &\approx -\ln x + \frac{1}{6}, \\
F_1(x_c) &\approx 8.58, \\
F_1(x_t) &\approx 0.550, \\
F_1(\infty) &= \frac{7}{24} \approx 0.292.
\end{aligned} \tag{27}$$

Formula (26) can be rewritten with good accuracy as:

$$H_{penguin} \approx \sqrt{2} G_F \sin \theta_C \cos \theta_C \left(-\frac{\alpha_s}{12\pi} \ln \frac{m_c^2}{\mu^2} + i \frac{\text{Im} V_{cd} V_{cs}^*}{\text{Re} V_{cd} V_{cs}^*} \frac{\alpha_s}{12\pi} \ln \frac{M_W^2}{m_c^2} \right) O_5, \tag{28}$$

where instead of m_u the characteristic hadronic scale μ (this time “low” normalization point) is substituted.

Thus the real and imaginary parts of c_5 are equal to:

$$\begin{aligned}
\text{Re} c_5 &= -\frac{\alpha_s}{12\pi} \ln \frac{m_c^2}{\mu^2} \\
\text{Im} c_5 &= \frac{\text{Im} V_{cd} V_{cs}^*}{\text{Re} V_{cd} V_{cs}^*} \frac{\alpha_s}{12\pi} \ln \frac{M_W^2}{m_c^2}.
\end{aligned} \tag{29}$$

In order to understand at which virtuality α_s should be taken in these expressions leading logarithms should be summed up. This was done for the real part of coefficient function in the paper [25]:

$$Rec_5 = \left(\chi_1^{0.48} \left(-0.039 \chi_2^{0.8} + 0.033 \chi_2^{0.42} + 0.003 \chi_2^{-0.12} + 0.003 \chi_2^{-0.3} \right) + \chi_1^{-0.24} \left(-0.014 \chi_2^{0.8} - 0.001 \chi_2^{0.42} - 0.014 \chi_2^{-0.12} + 0.029 \chi_2^{-0.3} \right) \right), \quad (30)$$

while for imaginary part in paper [26] the following result was obtained:

$$Imc_5 = \frac{ImV_{cd}V_{cs}^*}{ReV_{cd}V_{cs}^*} \left(0.0494 \chi_1^{0.85} - 0.0280 \chi_1^{0.42} + 0.0116 \chi_1^{-0.13} - 0.0330 \chi_1^{-0.35} \right) \times \left(0.8509 \chi_2^{0.8} + 0.0091 \chi_2^{0.42} + 0.1222 \chi_2^{-0.12} + 0.0178 \chi_2^{-0.3} \right), \quad (31)$$

where $\chi_1 = \frac{\alpha_s(m_c)}{\alpha_s(m_W)}$, $\chi_2 = \frac{\alpha_s(\mu)}{\alpha_s(m_c)}$.

Numerical analysis shows that with a good accuracy expression for Rec_5 can be written as:

$$Rec_5 = -\frac{\alpha_s(\mu)}{12\pi} \log\left(\frac{m_c^2}{\mu^2}\right). \quad (32)$$

On the other hand Imc_5 at the scale μ at which $\alpha_s(\mu) = 1$ has the value

$$Imc_5 = \frac{ImV_{cd}V_{cs}^*}{ReV_{cd}V_{cs}^*} \times 0.13. \quad (33)$$

The expression for ImA_0 can be written as:

$$ImA_0 = \sqrt{2}G_F \sin \theta_C \cos \theta_C Im(c_5) < (\pi\pi)_{I=0} | O_5 | K^0 > \quad (34)$$

In order to get the value of ImA_0 we must calculate hadronic matrix element of penguin operator. It was evaluated in the framework of naive quark model in [25], see also [27]:

$$< (\pi\pi)_{I=0} | O_5 | K^0 > = \frac{4\sqrt{6}}{9} \frac{m_K^2 m_\pi^2 f_\pi}{m_s(m_u + m_d)} \left(\frac{f_K}{f_\pi} \left(1 + \frac{m_K^2}{m_\sigma^2} \right) - 1 \right) \quad (35)$$

Substituting experimental numbers $G_F = 1.166 \times 10^{-5} GeV$, $\sin \theta_C = 0.22$, $\cos \theta_C = 0.95$, $m_\pi = 135 MeV$, $m_K = 497 MeV$, $f_\pi = 130 MeV$, $f_K = 160 MeV$, $m_\sigma = 700 MeV$ and quark masses $m_s = 130 MeV$, $m_u = 3 MeV$, $m_d = 7 MeV$, we get:

$$ImA_0 = -1.1 \times 10^{-10} GeV. \quad (36)$$

Finally, dividing it by experimentally measured $ReA_0 = 3.33 \times 10^{-7} GeV$, we obtain:

$$\frac{ImA_0}{ReA_0} = -3.2 \times 10^{-4}. \quad (37)$$

In order to estimate the theoretical error for ImA_0 we propose the following method: to take two values of μ , corresponding to $\alpha_s(\mu) = \frac{2}{3}$ and $\alpha_s(\mu) = \frac{3}{2}$, to calculate $\frac{ImA_0}{ReA_0}$ at each μ and from these boundary values get \pm error for $\frac{ImA_0}{ReA_0}$.

Via the proposed method we get our final result:

$$\frac{ImA_0}{ReA_0} = -(3.2^{+1.1}_{-0.8}) \times 10^{-4}. \quad (38)$$

This number is rather stable with respect to the variation of μ . Our result confirms statement made in [27]: QCD penguin results in the value of $\frac{\epsilon'}{\epsilon}$ in ballpark of the experimental data.

On the other hand the real part is very sensitive to μ , as can be seen from Eq.(32). The expression for ReA_0 has the following form:

$$ReA_0 = \sqrt{2}G_F \sin \theta_C \cos \theta_C Re(c_5) < (\pi\pi)_{I=0} | O_5 | K^0 > . \quad (39)$$

Substituting numbers and again taking μ at which $\alpha_s(\mu) = \frac{2}{3}$ and $\alpha_s(\mu) = \frac{3}{2}$ we get: $ReA_0 = (1.2^{+0.8}_{-0.6}) \times 10^{-7} GeV$. The central number is approximately 3 times smaller than the experimental, but theoretical uncertainty is large.

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